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A Simple Grand Unification View of Neutrino Mixing and Fermion Mass Matrices

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Abstract

Assuming three light neutrinos and the see-saw mechanism we present a semiquantitative model of fermion masses based on (SUSY) SU(5) and abelian horizontal charges. A good description of the observed pattern of quark and lepton masses is obtained. For neutrinos we naturally obtain widely split masses and large atmospheric neutrino mixing as a consequence of SU(5)-related asymmetric mass matrices for d quarks and charged leptons.

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Following the experimental results from Superkamiokande [1] a lot of attention has been devoted to the problem of a natural explanation of the observed nearly maximal mixing angle for atmospheric neutrino oscillations. It is possible that also solar neutrino oscillations occur with a large mixing angle [2]. Large mixing angles are somewhat unexpected because the observed quark mixings are small and the quark, charged lepton and neutrino mass matrices are to some extent related in Grand Unified Theories. The challenge is to incorporate the new information on neutrino mixings in a comprehensive picture of fermion masses. In previous papers [4, 5] we have given a general discussion of this problem and have proposed a class of solutions for a natural explanation of maximal mixing within the framework of the see-saw mechanism [6]. In this article we review our strategy and present some further examples of realizations of our approach in the context of Grand Unified Theories (GUT's).

The experimental status of neutrino oscillations is still very preliminary. While the evidence for the existence of neutrino oscillations from solar and atmospheric neutrino data is rather convincing by now, the values of the mass squared differences Δm^2 and mixing angles are not firmly established. For solar neutrinos, for example, three possible solutions are still possible [2]. Two are based on the MSW mechanism [7], one with small (MSW-SA) and one with large mixing angle (MSW-LA), and one in terms of vacuum oscillations (VO) with large mixing angle, with the MSW-LA solution being now somewhat less favoured than the other two [2, 3]. For atmospheric neutrinos the preferred value of Δm^2 is affected by large uncertainties and could still sizeably drift in one sense or the other, but the fact that the mixing angle is large appears established ($\sin^2 2\theta_{atm} \gtrsim 0.8$) [8, 9]. Another issue which is still open is the claim by the LNSD collaboration of an additional signal of neutrino oscillations in a reactor experiment [10]. This claim was not sofar supported by a second recent experiment, Karmen [11], which at face value contradicts the LNSD result, but the issue is far from being closed. Given the present experimental uncertainties the theorist has to make some assumptions on how the data will finally look like in the future. Here we tentatively assume that the LNSD evidence will disappear. If so then we only have two oscillations frequencies, which can be given in terms of the three known species of light neutrinos without additional sterile kinds. We then take for granted that the frequency of atmospheric neutrino oscillations will remain well separated from the solar neutrino frequency, even for the MSW solutions. The present best values are [2, 8, 9] $(\Delta m^2)_{atm} \sim 2 \cdot 10^{-3}\ eV^2$ and $(\Delta m^2)_{MSW-SA} \sim 5 \cdot 10^{-6}\ eV^2$ or $(\Delta m^2)_{VO} \sim 10^{-10}\ eV^2$. We also assume that the electron neutrino does not participate in the atmospheric oscillations, which (in absence of sterile neutrinos) are interpreted as nearly maximal $\nu_\mu \rightarrow \nu_\tau$ oscillations as indicated by the Superkamiokande [1] and Chooz [12] data. However the data do not exclude a non-vanishing U_{e3} element. In most of the Superkamiokande allowed region the bound by Chooz [12] amounts to $|U_{e3}| \lesssim 0.2$ but in the region not covered by Chooz $|U_{e3}|$ could even be somewhat larger [8, 9]. If we neglect CP violation phases and adopt a particular set of sign conventions, the neutrino mixing matrix U is then fixed by the above assumptions in the form explicitly given in ref. [5, 13] (see also [14]) in terms of the solar mixing angle (which can be either very small (MSW-SA: $\sin^2 2\theta_{sun} \sim 5.5 \cdot 10^{-3}$) or nearly maximal (VO: $\sin^2 2\theta_{sun} \sim 0.75$).

Neutrino oscillations imply neutrino masses which in turn demand either the existence of right-handed neutrinos or lepton number violation or both. Given that the neutrino masses are certainly extremely small, it is really difficult from the theory point of view to avoid the

conclusion that lepton number L must be violated. In fact it is only in terms of lepton number violation that the smallness of neutrino masses can be connected to the very large scale where L is violated, of order M_{GUT} or even $M_{Pl} \sim 2.4 \cdot 10^{18}$ GeV. If L is not conserved, even in the absence of ν_R, Majorana masses can be generated for neutrinos by dimension five operators of the form $O_5 = L_i^T \lambda_{ij} L_j \phi\phi/M$ with ϕ being the ordinary Higgs doublet, λ a matrix in flavour space and M a large scale of mass. However we consider that the existence of ν_R is quite plausible because all GUT groups larger than SU(5) require them. In particular the fact that ν_R completes the representation 16 of SO(10): 16=$\bar{5}$+10+1, so that all fermions of each family are contained in a single representation of the unifying group, is too impressive not to be significant. Thus in the following we assume that there are both ν_R and lepton number violation.

With these assumptions the see-saw mechanism is possible and the resulting neutrino mass matrix is of the form $L_i^T m_{\nu ij} L_j$ with $m_\nu = m_D^T M^{-1} m_D$ where m_D and M are the neutrino Dirac matrix, $\bar{R} m_D L$, and the Majorana matrix, $\bar{R} M \bar{R}^T$, respectively. Here we assume that the additional non renormalisable terms from O_5 are comparatively negligible, otherwise they should simply be added. After elimination of the heavy right-handed fields, at the level of the effective low energy theory, the two types of terms are equivalent. In particular they have identical transformation properties under a chiral change of basis in flavour space. The difference is, however, that in the see-saw mechanism, the Dirac matrix m_D is presumably related to ordinary fermion masses because they are both generated by the Higgs mechanism and both must obey GUT-induced constraints. Thus if we assume the see-saw mechanism more constraints are implied. In particular we are led to the natural hypothesis that m_D has a largely dominant third family eigenvalue in analogy to m_t, m_b and m_τ which are by far the largest masses among u quarks, d quarks and charged leptons. Once we accept that m_D is hierarchical it is very difficult to imagine that the effective light neutrino matrix, generated by the see-saw mechanism, could have eigenvalues very close in absolute value.

Since neutrino oscillations only measure differences of squared masses, the observed differences $(\Delta m^2)_{atm} = |m_3^2 - m_2^2| \gg (\Delta m^2)_{sun} = |m_2^2 - m_1^2|$ could correspond to A) hierarchical eigenvalues $|m_3| \gg |m_{2,1}|$ (that m_1 and m_2 are close or very different is irrelevant to our purposes) or to partial or total near degeneracy: B) $|m_1| \sim |m_2| \gg |m_3|$ or C) $|m_1| \sim |m_2| \sim |m_3|$ (the numbering 1,2,3 corresponds to our definition of the frequencies as in ref. [5] and in principle may not coincide with the family index although this will be the case in the models that we favour). The configurations B) and C) imply a very precise near degeneracy of squared masses. For example, the case C) is the only one that could in principle accommodate neutrinos as hot dark matter together with solar and atmospheric neutrino oscillations. We think that it is not at all clear at the moment that a hot dark matter component is really needed [15] but this could be a reason in favour of the fully degenerate solution. Then the common mass should be around 1-3 eV. The solar frequency could be given by a small 1-2 splitting, while the atmospheric frequency could be given by a still small but much larger 1,2-3 splitting. A strong constraint arises in this case from the non observation of neutrinoless double beta decay which requires that the ee entry of m_ν must obey $|(m_\nu)_{ee}| \leq 0.46$ eV [16]. As observed in ref. [17], this bound can only be satisfied if bimixing is realized (that is double maximal mixing, with solar neutrinos explained by the VO or MSW-LA solutions). But we would need

a relative splitting $|\Delta m/m| \sim \Delta m^2_{atm}/2m^2 \sim 10^{-3} - 10^{-4}$ and a much smaller one for solar neutrinos explained by vacuum oscillations: $|\Delta m/m| \sim 10^{-10} - 10^{-11}$. As mentioned above we consider it unplausible that starting from hierarchical Dirac matrices we end up via the see-saw mechanism into a nearly perfect degeneracy of squared masses. In conclusion the assumption of hierarchical Dirac masses and the see-saw mechanism naturally leads to a pattern of type A with $|m_3| \gg |m_2| \gg |m_1|$. Models with degenerate neutrinos (see, for example, refs. [18]) could be natural if the dominant contributions directly arise from non renormalisable operators like O_5 which are apriori unrelated to other fermion masses, but we will not explore this possibility here.

Thus we are led to consider models with large effective light neutrino mass splittings and large mixings. In general large splittings correspond to small mixings because normally only close-by states are strongly mixed. In a 2 by 2 matrix context the requirement of large splitting and large mixings leads to a condition of vanishing determinant. For example the matrix

$$m \propto \begin{bmatrix} x^2 & x \\ x & 1 \end{bmatrix} \tag{1}$$

has eigenvalues 0 and $1 + x^2$ and for x of 0(1) the mixing is large. Thus in the limit of neglecting small mass terms of order $m_{1,2}$ the demands of large atmospheric neutrino mixing and dominance of m_3 translate into the condition that the 2 by 2 subdeterminant 23 of the 3 by 3 mixing matrix vanishes. The problem is to show that this vanishing can be arranged in a natural way without fine tuning. We have discussed suitable possible mechanisms in our previous paper [5]. We in particular favour a class of models where, in the limit of neglecting terms of order $m_{1,2}$ and <u>in the basis where charged leptons are diagonal</u>, the Dirac matrix m_D, defined by $\bar{R} m_D L$, takes the approximate form:

$$m_D \propto \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & x & 1 \end{bmatrix} \quad . \tag{2}$$

This matrix has the property that for a generic Majorana matrix M one finds:

$$m_\nu = m_D^T M^{-1} m_D \propto \begin{bmatrix} 0 & 0 & 0 \\ 0 & x^2 & x \\ 0 & x & 1 \end{bmatrix} \quad . \tag{3}$$

The only condition on M^{-1} is that the 33 entry is non zero. It is important for the following discussion to observe that m_D given by eq. (2) under a change of basis transforms as $m_D \rightarrow V^\dagger m_D U$ where V and U rotate the right and left fields respectively. It is easy to check that in order to make m_D diagonal we need large left mixings. More precisely m_D is diagonalized by taking V=1 and U given by

$$U = \begin{bmatrix} c & -s & 0 \\ s c_\gamma & c c_\gamma & -s_\gamma \\ s s_\gamma & c s_\gamma & c_\gamma \end{bmatrix} \quad , \tag{4}$$

with

$$s_\gamma = -x/r \quad , \qquad c_\gamma = 1/r \quad , \qquad r = \sqrt{1 + x^2} \quad . \tag{5}$$

The matrix U is directly the neutrino mixing matrix. The mixing angle for atmospheric neutrino oscillations is given by:

$$\sin^2 2\theta = 4s_\gamma^2 c_\gamma^2 = \frac{4x^2}{(1+x^2)^2} \quad . \tag{6}$$

Thus the bound $\sin^2 2\theta \gtrsim 0.8$ translates into $0.6 \lesssim |x| \lesssim 1.6$. It is interesting to recall that in refs. [19, 20] it was shown that the mixing angle can be amplified by the running from a large mass scale down to low energy.

We have seen that, in order to explain in a natural way widely split light neutrino masses together with large mixings, we need an automatic vanishing of the 23 subdeterminant. This in turn is most simply realized by allowing some large left-handed mixing terms in the Dirac neutrino matrix. By left-handed mixing we mean non diagonal matrix elements that can only be eliminated by a large rotation of the left-handed fields. Thus the question is how to reconcile large left-handed mixings in the leptonic sector with the observed near diagonal form of V_{CKM}, the quark mixing matrix. Strictly speaking, since $V_{CKM} = U_u^\dagger U_d$, the individual matrices U_u and U_d need not be near diagonal, but V_{CKM} does, while the analogue for leptons apparently cannot be near diagonal. However nothing forbids for quarks that, in the basis where m_u is diagonal, the d quark matrix has large non diagonal terms that can be rotated away by a pure right-handed rotation. We suggest that this is so and that in some way right-handed mixings for quarks correspond to left-handed mixings for leptons.

In the context of (Susy) SU(5) [21] there is a very attractive hint of how the present mechanism can be realized. In the $\bar{5}$ of SU(5) the d^c singlet appears together with the lepton doublet (ν, e). The (u, d) doublet and e^c belong to the 10 and ν^c to the 1 and similarly for the other families. As a consequence, in the simplest model with mass terms arising from only Higgs pentaplets, the Dirac matrix of down quarks is the transpose of the charged lepton matrix: $m_D^d = (m_D^l)^T$. Thus, indeed, a large mixing for right-handed down quarks corresponds to a large left-handed mixing for charged leptons. In the same simplest approximation with 5 or $\bar{5}$ Higgs, the up quark mass matrix is symmetric, so that left and right mixing matrices are equal in this case [1]. Then small mixings for up quarks and small left-handed mixings for down quarks are sufficient to guarantee small V_{CKM} mixing angles even for large d quark right-handed mixings. When the charged lepton matrix is diagonalized the large left-handed mixing of the charged leptons is transferred to the neutrinos. Note that in SU(5) we can diagonalize the u mass matrix by a rotation of the fields in the 10, the Majorana matrix M by a rotation of the 1 and the effective light neutrino matrix m_ν by a rotation of the $\bar{5}$. In this basis the d quark mass matrix fixes V_{CKM} and the charged lepton mass matrix fixes neutrino mixings. It is well known that a model where the down and the charged lepton mass matrices are exactly the transpose of one another cannot be exactly true because of the e/d and μ/s mass ratios [21]. It is also known that one remedy to this problem is to add some Higgs component in the 45 representation of SU(5) [22]. A different solution [23] will be described later. But the symmetry under transposition can still be a good guideline if we are only interested in the order of magnitude of the matrix entries and not in their exact values. Similarly, the Dirac neutrino mass matrix m_D is the same as the up quark mass matrix in the very crude model where the Higgs pentaplets

[1] Up to a diagonal matrix of phases.

come from a pure 10 representation of SO(10): $m_D = m_D^u$. For m_D the dominance of the third family eigenvalue as well as a near diagonal form could be an order of magnitude remnant of this broken symmetry. Thus, neglecting small terms, the neutrino Dirac matrix in the basis where charged leptons are diagonal could be directly obtained in the form of eq. (2).

We give here an explicit example of the mechanism under discussion in the framework of a unified Susy $SU(5)$ theory with an additional $U(1)_F$ flavour symmetry [24]. This model is to be taken as merely indicative, in that some important problems, like, for example, the cancellation of chiral anomalies are not tackled here. But we find it impressive that the general pattern of all what we know on fermion masses and mixings is correctly reproduced at the level of orders of magnitude. We regard the present model as a low-energy effective theory valid at energies close to $M_{GUT} \ll M_{Pl}$. We can think to obtain it by integrating out the heavy modes from an unknown underlying fundamental theory defined at an energy scale close to M_{Pl}. From this point of view the gauge anomalies generated by the light supermultiplets listed below can be compensated by another set of supermultiplets with masses above M_{GUT}, already eliminated from the low-energy theory. In particular, we assume that these additional supermultiplets are vector-like with respect to $SU(5)$ and chiral with respect to $U(1)_F$. Their masses are then naturally expected to be of the order of the $U(1)_F$ breaking scale, which, in the following discussion, turns out to be near M_{Pl}. We have explicitly checked the possibility of canceling the gauge anomalies in this way but, due to our ignorance about the fundamental theory, we do not find particularly instructive to illustrate the details here. In this model the known generations of quarks and leptons are contained in triplets Ψ_{10}^a and $\Psi_{\bar{5}}^a$, $(a = 1, 2, 3)$ transforming as 10 and $\bar{5}$ of $SU(5)$, respectively. Three more $SU(5)$ singlets Ψ_1^a describe the right-handed neutrinos. We assign to these fields the following F-charges:

$$\Psi_{10} \sim (3, 2, 0) \tag{7}$$

$$\Psi_{\bar{5}} \sim (3, 0, 0) \tag{8}$$

$$\Psi_1 \sim (1, -1, 0) \tag{9}$$

We start by discussing the Yukawa coupling allowed by $U(1)_F$-neutral Higgs multiplets φ_5 and $\varphi_{\bar{5}}$ in the 5 and $\bar{5}$ $SU(5)$ representations and by a pair θ and $\bar{\theta}$ of $SU(5)$ singlets with $F = 1$ and $F = -1$, respectively.

In the quark sector we obtain [2]:

$$m_D^u = (m_D^u)^T = \begin{bmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix} v_u \ , \qquad m_D^d = \begin{bmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{bmatrix} v_d \ , \tag{10}$$

from which we get the order-of-magnitude relations:

$$\begin{aligned} m_u : m_c : m_t &= \lambda^6 : \lambda^4 : 1 \\ m_d : m_s : m_b &= \lambda^6 : \lambda^2 : 1 \end{aligned} \tag{11}$$

[2] In eq. (10) the entries denoted by 1 in m_D^u and m_D^d are not necessarily equal. As usual, such a notation allows for O(1) deviations.

and

$$V_{us} \sim \lambda \ , \qquad V_{ub} \sim \lambda^3 \ , \qquad V_{cb} \sim \lambda^2 \ . \tag{12}$$

Here $v_u \equiv \langle \varphi_5 \rangle$, $v_d \equiv \langle \varphi_{\bar{5}} \rangle$ and λ denotes the ratio between the vacuum expectation value of $\bar{\theta}$ and an ultraviolet cut-off identified with the Planck mass M_{Pl}: $\lambda \equiv \langle \bar{\theta} \rangle / M_{Pl}$. To correctly reproduce the observed quark mixing angles, we take λ of the order of the Cabibbo angle. For non-negative F-charges, the elements of the quark mixing matrix V_{CKM} depend only on the charge differences of the left-handed quark doublet [24]. Up to a constant shift, this defines the choice in eq. (7). Equal F-charges for $\Psi_{\bar{5}}^{2,3}$ (see eq. (8)) are then required to fit m_b and m_s. We will comment on the lightest quark masses later on.

At this level, the mass matrix for the charged leptons is the transpose of m_D^d:

$$m_D^l = (m_D^d)^T \tag{13}$$

and we find:

$$m_e : m_\mu : m_\tau = \lambda^6 : \lambda^2 : 1 \tag{14}$$

The O(1) off-diagonal entry of m_D^l gives rise to a large left-handed mixing in the 23 block which corresponds to a large right-handed mixing in the d mass matrix. In the neutrino sector, the Dirac and Majorana mass matrices are given by:

$$m_D = \begin{bmatrix} \lambda^4 & \lambda & \lambda \\ \lambda^2 & \lambda' & \lambda' \\ \lambda^3 & 1 & 1 \end{bmatrix} v_u \ , \qquad M = \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & \lambda'^2 & \lambda' \\ \lambda & \lambda' & 1 \end{bmatrix} \bar{M} \ , \tag{15}$$

where $\lambda' \equiv \langle \theta \rangle / M_{Pl}$ and $\bar{M}$ denotes the large mass scale associated to the right-handed neutrinos: $\bar{M} \gg v_{u,d}$.

After diagonalization of the charged lepton sector and after integrating out the heavy right-handed neutrinos we obtain the following neutrino mass matrix in the low-energy effective theory:

$$m_\nu = \begin{bmatrix} \lambda^6 & \lambda^3 & \lambda^3 \\ \lambda^3 & 1 & 1 \\ \lambda^3 & 1 & 1 \end{bmatrix} \frac{v_u^2}{\bar{M}} \tag{16}$$

where we have taken $\lambda \sim \lambda'$. The O(1) elements in the 23 block are produced by combining the large left-handed mixing induced by the charged lepton sector and the large left-handed mixing in m_D. A crucial property of m_ν is that, as a result of the sea-saw mechanism and of the specific $U(1)_F$ charge assignment, the determinant of the 23 block is *automatically* of $O(\lambda^2)$ (for this the presence of negative charge values, leading to the presence of both λ and λ' is essential [5]).

It is easy to verify that the eigenvalues of m_ν satisfy the relations:

$$m_1 : m_2 : m_3 = \lambda^4 : \lambda^2 : 1 \ \ . \tag{17}$$

The atmospheric neutrino oscillations require $m_3^2 \sim 10^{-3}$ eV^2. From eq. (16), taking $v_u \sim$ 250 GeV, the mass scale $\bar{M}$ of the heavy Majorana neutrinos turns out to be close to the

unification scale, $\bar{M} \sim 10^{15}$ GeV. The squared mass difference between the lightest states is of $O(\lambda^4)\ m_3^2$, appropriate to the MSW solution to the solar neutrino problem. Finally, beyond the large mixing in the 23 sector, corresponding to $s_\gamma \sim c_\gamma$ in eq. (4), m_ν provides a mixing angle $s \sim (\lambda/2)$ in the 12 sector, close to the range preferred by the small angle MSW solution. In general U_{e3} is non-vanishing, of $O(\lambda^3)$.

In general, the charge assignment under $U(1)_F$ allows for non-canonical kinetic terms that represent an additional source of mixing. Such terms are allowed by the underlying flavour symmetry and it would be unnatural to tune them to the canonical form. We have checked that all the results quoted up to now remain unchanged after including the effects related to the most general kinetic terms, via appropriate rotations and rescaling in the flavour space (see also ref.[25]).

Obviously, the order of magnitude description offered by this model is not intended to account for all the details of fermion masses. Even neglecting the parameters associated with the CP violating observables, some of the relevant observables are somewhat marginally reproduced. For instance we obtain $m_u/m_t \sim \lambda^6$ which is perhaps too large. However we find it remarkable that in such a simple scheme most of the 12 independent fermion masses and the 6 mixing angles turn out to have the correct order of magnitude. Notice also that our model prefers large values of $\tan\beta \equiv v_u/v_d$. This is a consequence of the equality $F(\Psi_{10}^3) = F(\Psi_{\bar{5}}^3)$ (see eqs. (7) and (8)). In this case the Yukawa couplings of top and bottom quarks are expected to be of the same order of magnitude, while the large m_t/m_b ratio is attributed to $v_u \gg v_d$ (there may be factors O(1) modifying these considerations, of course). We recall here that in supersymmetric grand unified models large values of $\tan\beta$ are one possible solution to the problem of reconciling the boundary condition $m_b = m_\tau$ at the GUT scale with the low-energy data [26]. Alternatively, to keep $\tan\beta$ small, one could suppress m_b/m_t by adopting different F-charges for the $\Psi_{\bar{5}}^3$ and Ψ_{10}^3.

Additional contributions to flavour changing processes and to CP violating observables are generally expected in a supersymmetric grand unified theory. However, a reliable estimate of the corresponding effects would require a much more detailed definition of the theory than attempted here. Crucial ingredients such as the mechanism of supersymmetry breaking and its transmission to the observable sector have been ignored in the present note. We are implicitly assuming that the omission of this aspect of the flavour problem does not substantially alter our discussion.

A common problem of all $SU(5)$ unified theories based on a minimal higgs structure is represented by the relation $m_D^l = (m_D^d)^T$ that, while leading to the successful $m_b = m_\tau$ boundary condition at the GUT scale, provides the wrong prediction $m_d/m_s = m_e/m_\mu$ (which, however, is an acceptable order of magnitude equality). We can easily overcome this problem and improve the picture [23] by introducing an additional supermultiplet $\bar{\theta}_{24}$ transforming in the adjoint representation of $SU(5)$ and possessing a negative $U(1)_F$ charge, $-n\ \ (n > 0)$. Under these conditions, a positive F-charge f carried by the matrix elements $\Psi_{10}^a \Psi_{\bar{5}}^b$ can be compensated in several different ways by monomials of the kind $(\bar{\theta})^p(\bar{\theta}_{24})^q$, with $p + nq = f$. Each of these possibilities represents an independent contribution to the down quark and charged

lepton mass matrices, occurring with an unknown coefficient of O(1). Moreover the product $(\bar{\theta}_{24})^q \varphi_{\bar{5}}$ contains both the $\bar{5}$ and the $\overline{45}$ $SU(5)$ representations, allowing for a differentiation between the down quarks and the charged leptons. The only, welcome, exceptions are given by the O(1) entries that do not require any compensation and, at the leading order, remain the same for charged leptons and down quarks. This preserves the good $m_b = m_\tau$ prediction. Since a perturbation of O(1) in the subleading matrix elements is sufficient to cure the bad $m_d/m_s = m_e/m_\mu$ relation, we can safely assume that $\langle\bar{\theta}_{24}\rangle/M_{Pl} \sim \lambda^n$, to preserve the correct order-of-magnitude predictions in the remaining sectors.

We have not dealt here with the problem of recovering the correct vacuum structure by minimizing the effective potential of the theory. It may be noticed that the presence of two multiplets θ and $\bar{\theta}$ with opposite F charges could hardly be reconciled, without adding extra structure to the model, with a large common VEV for these fields, due to possible analytic terms of the kind $(\theta\bar{\theta})^n$ in the superpotential [3]. We find therefore instructive to explore the consequences of allowing only the negatively charged $\bar{\theta}$ field in the theory.

It can be immediately recognized that, while the quark mass matrices of eqs. (10) are unchanged, in the neutrino sector the Dirac and Majorana matrices get modified into:

$$m_D = \begin{bmatrix} \lambda^4 & \lambda & \lambda \\ \lambda^2 & 0 & 0 \\ \lambda^3 & 1 & 1 \end{bmatrix} v_u \quad , \qquad M = \begin{bmatrix} \lambda^2 & 1 & \lambda \\ 1 & 0 & 0 \\ \lambda & 0 & 1 \end{bmatrix} \bar{M} \quad . \tag{18}$$

The zeros are due to the analytic property of the superpotential that makes impossible to form the corresponding F invariant by using $\bar{\theta}$ alone. These zeros should not be taken literally, as they will be eventually filled by small terms coming, for instance, from the diagonalization of the charged lepton mass matrix and from the transformation that put the kinetic terms into canonical form. It is however interesting to work out, in first approximation, the case of exactly zero entries in m_D and M, when forbidden by F.

The neutrino mass matrix obtained via see-saw from m_D and M has the same pattern as the one displayed in eq. (16). A closer inspection reveals that the determinant of the 23 block is identically zero, independently from λ. This leads to the following pattern of masses:

$$m_1 : m_2 : m_3 = \lambda^3 : \lambda^3 : 1 \quad , \qquad m_1^2 - m_2^2 = \mathrm{O}(\lambda^9)\; m_3^2 \quad . \tag{19}$$

Moreover the mixing in the 12 sector is almost maximal:

$$\frac{s}{c} = \frac{\pi}{4} + \mathrm{O}(\lambda^3) \quad . \tag{20}$$

For $\lambda \sim 0.2$, both the squared mass difference $(m_1^2 - m_2^2)/m_3^2$ and $\sin^2 2\theta_{sun}$ are remarkably close to the values required by the vacuum oscillation solution to the solar neutrino problem. We have also checked that this property is reasonably stable against the perturbations induced by small terms (of order λ^5) replacing the zeros, coming from the diagonalization of the charged lepton sector and by the transformations that render the kinetic terms canonical. We find quite

[3] We thank N. Irges for bringing our attention on this point.

interesting that also the just-so solution, requiring an intriguingly small mass difference and a bimaximal mixing, can be reproduced, at least at the level of order of magnitudes, in the context of a "minimal" model of flavour compatible with supersymmetric SU(5). In this case the role played by supersymmetry is essential, a non-supersymmetric model with $\bar{\theta}$ alone not being distinguishable from the version with both θ and $\bar{\theta}$, as far as low-energy flavour properties are concerned.

Finally, let us compare our model with other recent proposals [27]. Textures for the effective neutrino mass matrix similar to m_ν in eq. (16) were derived in refs. [28, 29], also in the context of an $SU(5)$ unified theory with a $U(1)$ flavour symmetry. In these works, however, the O(1) entries of m_ν are uncorrelated, due to the particular choice of U(1) charges. The diagonalization of such matrix, for generic O(1) coefficients, leads to only one light eigenvalue and to two heavy eigenvalues, of O(1), in units of $v_u^2/\bar{M}$. Then the required pattern $m_3 \gg m_2 \sim m_1$ has to be fixed by hand. On the contrary, in our model the desired pattern is automatic, since, as emphasized above, the determinant of the 23 block in m_ν is vanishing at the leading order. Other models in terms of U(1) horizontal charges have been proposed in refs. [30, 31, 19]. Clearly a large mixing for the light neutrinos can be provided in part by the diagonalization of the charged lepton sector. As we have seen, in $SU(5)$, the left-handed mixing carried by charged leptons is expected to be, at least in first approximation, directly linked to the right-handed mixing for the d quarks and, as such, perfectly compatible with the available data. This possibility was remarked, for instance, in refs. [32, 33, 34, 20] where the implementation was in terms of asymmetric textures, of the Branco et al. type [35], used as a general parametrization of the existing data consistent with the constraints imposed by the unification program. On the other hand, our model aims to a dynamical explanation of the flavour properties, although in a simplified setting [4].

Our conclusion is that if we start from three light neutrinos and the see-saw mechanism then the most natural interpretation of the present data on neutrino oscillations is in terms of hierarchical light neutrino masses and asymmetric mass matrices (at least for d quarks and charged leptons). As well known, asymmetric matrices allow to reproduce the experimental value of $|V_{cb}|$ better than in the symmetric Fritzsch texture [36]. While SO(10), perhaps realized in some form at M_{Pl}, appears as a good classification group, with each family perfectly accommodated in a 16 representation, the description at M_{GUT} is more accurately formulated in terms of $SU(5)$. In this framework it is natural to obtain large splittings and large mixing angles for light neutrinos.

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